

PHILOSOPHY 8500

SEMINAR IN LOGIC: CONTINUITY AND INFINITY

SPRING 2022

We focus on two contrasts, one between the finite and the infinite, and the other between the continuous and the discrete.

Aristotle insisted that the actual infinite is incoherent, and that the only sensible notion is that of the potentially infinite, a process that can be continued indefinitely. This Aristotelian theme was echoed throughout the history of philosophy and mathematics, by, for example, Descartes, Leibniz, and Gauss. Today, thanks to the pioneering work of Cantor, the prevailing (but not universal) attitude is almost the diametric opposite, that the potentially infinite only makes sense if there is an actual infinity underlying it. But there are dissenting voices, from the traditional mathematical intuitionists and others.

Aristotle also argues that a continuous thing, such as a line segment (or a stretch of time), is not composed of points (or instants). Indeed, any part of a continuous thing is itself continuous. For Aristotle, points are only the boundaries of line segments (or potential line segments). Points are not *parts* of lines. This thesis, too, was echoed throughout history, with notable exceptions here and there. Today, the prevailing view is the diametric opposite: a line segment is no more than a set of points. But here, too, there are exceptions, serious theories of continuity that maintain at least some of the Aristotelian intuitions, in some cases at the cost of employing a non-classical logic.

A theme that connects our two contrasts is whether there are extended, infinitesimal line segments. Aristotle insisted that there are not, and this is reflected in Euclid's *Elements*. The standard contemporary treatments of the calculus also eschew infinitesimals. Cantor rejected them vehemently. But infinitesimals had an important role in the development of the calculus, and they are alive and well in alternate theories of the continuum—theories that enjoy a solid, rigorous foundation.

The only prerequisite is Philosophy 5500 (or equivalent). The course will be historical, metaphysical, and logical (and possibly value-laden). We will begin with Zeno's paradoxes, and Aristotle's response. After a look at some medieval thoughts on the topics, we will turn to the development of the calculus, mostly Descartes, Leibniz, and the trenchant critique of infinitesimals due to Berkeley. Then we will cover some contemporary theories of continuity: the dominant theory, due to Dedekind and Cantor (and others), and some alternatives.

Each enrolled student will write a few (very) short essays on various topics, a seminar paper that they will present to the class, a commentary on another student's seminar paper, and a medium-sized term paper. For distribution, the default is that the seminar will count as LLS. However, History or M&E credit can be arranged, if the student requests this early enough in the term and focuses his or her work in that area.