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Author: Eric Snyder

Title: A New Solution to Frege's Other Puzzle

Abstract: Hofweber (2005, 2007) presents two puzzles related to Frege's famous pair in (1a,b):

(1) a. Jupiter has four moons
b. The number of Jupiter's moons is four.
c.?? Jupiter has the number of Jupiter's moons moons. [Frege's Other Puzzle]
d. There is a number which is the number of Jupiter's moons, namely four. [Easy Argument]

The first puzzle has to do with the failure of substitutivity of the apparently coreferential singular term 'the number of Jupiter's moons' for 'four' in (1a), resulting in (1c). How can what appears to be one and the same expression 'four' have the opposing semantic functions of referring and quantifying? Hofweber calls this *Frege's Other Puzzle*. The second has to do with the apparently sound inference from (1a) and (1b) to (1d). How can we validly infer a clearly controversial metaphysical thesis – that numbers exist – from what appears to be a seemingly innocent observation about Jupiter's moons – that they are four in number? We might call this *the Easy Argument for Numbers*.

I want to present a new solution to both puzzles. The solution to Frege's Other Puzzle, I claim, is Partee-style type-shifting. According to Partee (1986), 'four' enters the lexicon as a predicative intersective adjective. It shifts up to an attributive adjective in examples like 'Every four boys formed a band', and this attributive adjective combines with another of Partee's type-shifters to get a determiner-type appropriate for 'Jupiter has four moons'. The main question is how to get referential uses as with 'Four is even', something Partee does not address. I suggest there is a type-lowering principle which takes the default cardinality predicate and returns the corresponding cardinal number, and that this type-lowering principle can be understood as a special case of a more general type-lowering principle which takes a degree predicate like 'four feet long' and returns a name for a degree like 'three feet (of length)'. On the resulting semantics, (1a) receives lower-bounded truth-conditions, and (1b) naturally receives two-sided truth-conditions. The solution to the Easy Argument is then that (1b) is not a meaning-preserving paraphrase of (1a), and so the argument is invalid. The solution to Frege's Other Puzzle is that there is no single lexical item 'four' serving the opposing semantic functions of referring, predicating, modifying, quantifying, etc. Rather, there are several lexical items 'four' corresponding to the different lexical categories witnessed, and these various lexical items are semantically related thanks to being lexically generated via type-shifting.

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Author: Chris Kennedy

Title: Numerals don't Form Horn Scales

Abstract: Consider the following sentences, uttered by the flight attendant in charge of explaining the safety features of the airplane in question:

(1) a. This airplane has five emergency exits.
b. This airplane has more than than four emergency exits.
c. This airplane has at least four emergency exits.

If the flight attendant utters (1a), we would usually take him to mean that the airplane has exactly five emergency exits, even though the sentence is consistent with the airplane having six or more emergency exits. This "two-sided" meaning is usually explained as a scalar implicature: 'five' and 'six' (and higher values) form a Horn scale; sentences with higher numerals entail the corresponding sentences with lower ones; therefore utterance of a sentence with a lower numeral licenses the inference that the corresponding sentences with higher numerals are false.

If the flight attendant utters (1b), we might wonder why he isn't giving us more precise information, but would conclude that the number of emergency exits is five or greater. We would NOT, however, understand the flight attendant to be conveying the same information as in (1a), which turns out to be somewhat surprising given that we could just as well apply the same reasoning to (1b) that we applied to (1a): 'more than five' ('more than six', etc.) asymmetrically entails 'more than four', so an utterance of the weaker 'more than four' ought to implicate that the stronger sentences are false. But it does not.

Finally, if the flight attendant utters (1c), we should start to think about getting off the plane, because this utterance --- unlike (1b) (or (1a)) --- strongly implicates that the flight attendant does not know how many emergency exits there are. At the same time, there is no upper-bounding inference here, even though once again the reasoning that we used to capture the two-sided meaning of (1a) should apply.

My goal in this talk is to present a semantic and pragmatic analysis of modified and unmodified numerals that: 1) derives ignorance implicatures of 'at least' and other superlative modifiers, 2) does not predict corresponding ignorance inferences for 'more than' and other comparative modifiers, and 3) correctly blocks upper-bounding inferences with both classes of modified numerals. The pragmatic component will be orthodox neo-Gricean, but in order to preserve orthodoxy, I will have to commit a bit of neo-Gricean heresy, and argue that numerals do not form Horn scales. This result, however, will follow from a new, "de-Fregean" semantics for numerals as second order properties of quantities, in which the two-sided meaning of (1a) is semantic rather than pragmatic.

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Author: Friederike Moltmann

Title: Number Terms as Proper Names

Abstract: In this talk, I will review my arguments for number terms being nonreferential in Moltmann (2013a, b) within the greater perspective of (my) recent work on proper names. I will argue that some of those arguments have to be reinterpreted: they show that number terms are mass (or number-neutral) terms in certain languages rather than showing their nonreferentiality.

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Author: Neil Tennant

Title: Referring to Reals

Abstract: I shall argue that our concept of real number is of geometric, or logico-geometric, provenance. It derives in a logically articulable way from our conception of the magnitude (of lengths) of straight-line segments, in terms of a chosen unit of length. The idea is fundamentally Euclidean. And it is intuitive rather than logical.

The nature of the reals can be brought out by a reconstructive 'logico-genetic' account. The account satisfies four conditions of adequacy that must be satisfied by any account of the reals. One of these adequacy conditions involves canonical identity statements involving two different ways of referring to reals. (This is the part that should interest the linguists.) I call the form of these identity statements Schema R. (It is analogous to what I called Schema N for natural numbers.)

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Author: Stewart Shapiro

Title: Frege on the Real Numbers

Abstract: This paper is concerned with Gottlob Frege's theory of the real numbers as sketched in the second volume of his masterpiece Grundgesetze der Arithmetik. It is clear that Frege's incomplete sketch represents a mathematically significant proposal in its own right, one which purports to have several important advantages over competing contemporary theories. It is perhaps unsurprising that Frege's theory of the real numbers is intimately intertwined with and largely motivated by his metaphysics, something which has of course received a great deal of independent attention. One of Frege's more significant claims in the Grundgesetze is that the cardinal numbers and the real numbers are ontologically distinct, or constitute "completely different domains". Cardinal numbers answer the question "How many things of a certain kind are there?", while real numbers answer the question "How large is a certain magnitude compared to a unit of magnitude of that same kind?" The account raises interesting, and surprisingly underexplored, questions about Frege's metaphysics: Can this metaphysics even accommodate mass quantities like water, gold, light intensity, or charge? Frege's main complaint with his contemporaries Cantor and Dedekind is that their theories of the real numbers do not build the applicability of the real numbers directly into the construction. In taking Cantor and Dedekind's Arithmetic theories to be insufficient, clearly Frege takes it to be a desideratum on a theory of the real numbers that their applicability be essential to their construction. But why? After all, it's not as if we can actually measure magnitudes like weight or density with the kind of infinite precision embodied by the real numbers anyway.

We begin with a detailed review of Frege's theory, one that mirrors Frege's exposition in structure. This is followed by a critique, outlining Frege's linguistic motivation for ontologically distinguishing the cardinal numbers from the real numbers. We briefly consider how Frege's metaphysics might need to be developed, or amended, to accommodate some of the problems. Finally, we offer a detailed examination of Frege's Application Constraint – that the reals ought to have their applicability built directly into their characterization. It bears on deeper questions concerning the relationship between sophisticated mathematical theories and their applications.

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Author: Paul Pietroski

Title: Conjunction, Subtraction, and Comparison

Abstract: In the talk, I'll offer some experimental evidence for the claim that 'most' is understood as a "number word," at least when it appears in combination with a plural count noun as in (1).

(1) Most of the dots are yellow.

An initially attractive idea, according to which 'most' is understood in terms of one to one correspondence—along with some other logical but not numeric vocabulary—turns out to be empirically implausible. But this leaves room for several possibilities regarding the "logical form" exhibited by sentences of the form 'Most of the Δs are Ψ'. I'll focus on (2) and (3), in part because Martin Hackl has offered independent arguments against (4).

(2) $\#(\Delta \;\&\; \Psi) > \#(\Delta \;\&\; {\sim}\Psi)$

(3) $\#(\Delta \;\&\; \Psi) > \#(\Delta) - \#(\Delta \;\&\; \Psi)$

(4) $\#(\Delta \;\&\; \Psi) > \#(\Delta)/2$

One can imagine Language Acquisition Devices that cannot represent *cardinalities*, or at least cannot represent cardinality *subtraction*. But it is independently plausible that human children, along with other animals, have the resources required to form instances of (3). This raises the question of whether adults end up understanding 'most' in a common way, and if so, which way. I'll argue that (3) is on the right track. The strategy involves showing that in suitably controlled settings, the ease/difficulty of evaluating (1)—deciding whether it is true or false—*does not reflect* the ease/difficulty of determining the number of dots that are not yellow, but *it does reflect* the ease/difficulty of determining the number of dots. As time permits, I'll also offer some data that bears on the analysis of sentences like (5),

(5) Most of the paint is yellow.

and a speculation about how *quant*ifiers can combine with mass nouns that are not disguised count nouns. Though here, I think the empirical findings point to the limitations of Frege's toolkit—designed to study the foundations of arithmetic—for the study of languages that children naturally acquire.

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Author: Thomas Hofweber

Title: The Semantics of Number Words and the Philosophy of Arithmetic

Abstract: The semantics of number words in natural language is of special interest for the philosophy of arithmetic. I hope to show which semantic questions are (and aren't) central for the philosophy of arithmetic, what the answer to these questions should be taken to be, and what position in the philosophy of arithmetic is supported by these answers.

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